Annual Concrete Seminar 2015

Youth Square

Mechanical Properties of Local Concrete

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Introduction

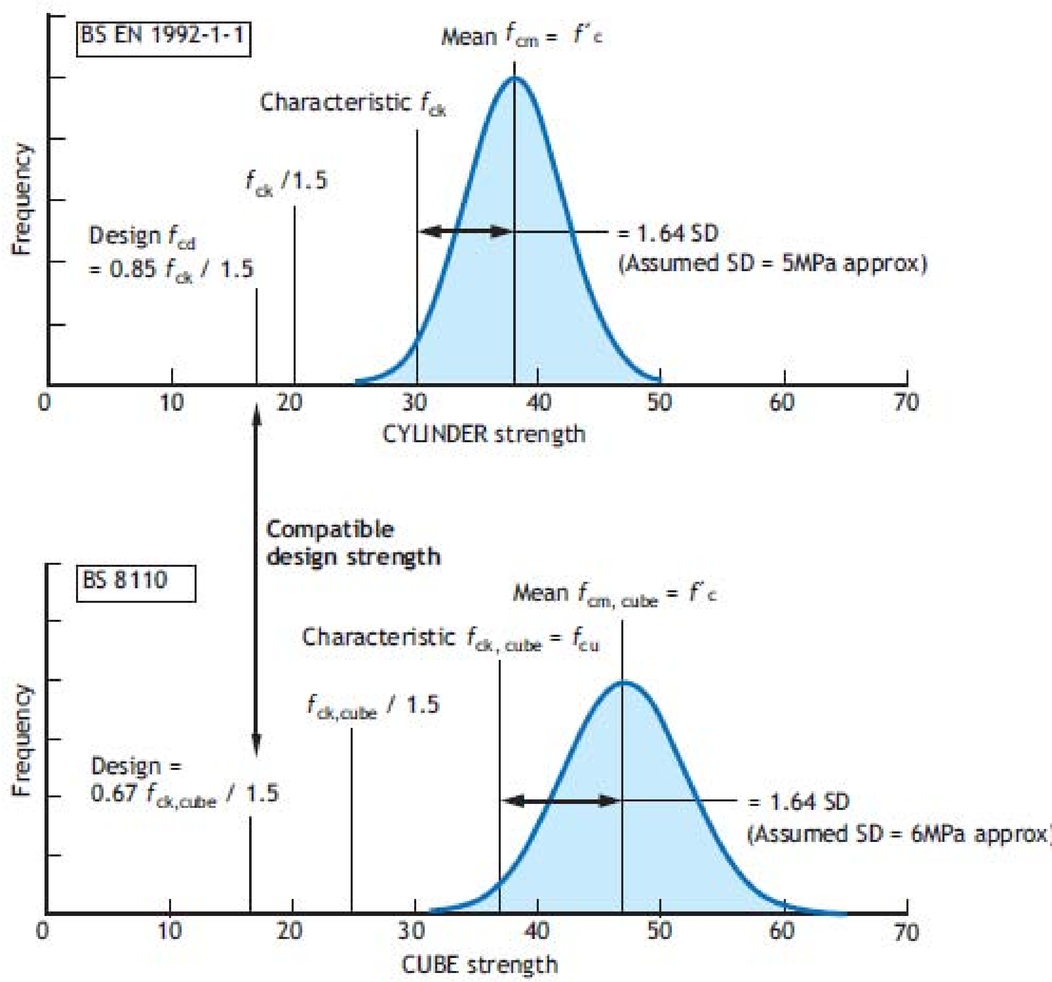
- The mechanical properties of concrete are dependent on many factors, such as the characteristics of the raw materials used, the curing conditions and the local concrete practice, which may vary from place to place.
- The mechanical properties measured in other places cannot be directly applicable to Hong Kong.
- To derive design models more suitable for use in Hong Kong, separate studies on the mechanical properties of concrete made in Hong Kong under the local conditions should be conducted.

Mechanical Properties of Local Concrete

- Mean, characteristic and design compressive strengths
- Full range stress-strain curve
- Initial elastic modulus
- Tensile strength
- Tension softening curve
- Maximum shear strength

Concrete Compressive Strength

Mean strength = Characteristic strength + 1.64 SD



Cylinder Strength

Eurocode
SDM2013

SD = 5 MPa approx.

Cube Strength

British Standards
CoP SUC2013

SD = 6 MPa approx.

Compressive Strength Definitions to Eurocode and British Standard for Strength Class C30/C37

Source: Bamforth et al. (2008)

Concrete Compressive Strength

Review of concrete cube strength from local concrete

Grade	$f_{cu,d}$ (MPa)	Samples	SD (MPa)	$f_{cu,m}$ (MPa)	$f_{cu,k}$ (MPa)	$f_{cu,k}$-$f_{cu,d}$ (MPa)	$f_{cu,m}/f_{cu,d}$ (MPa)
C30	30	1964	5.3	49	40.3	**10.3**	**1.63**
C35	35	12444	5.2	54	45.5	**10.5**	**1.54**
C40	40	1886	6.9	60	48.7	**8.70**	**1.50**
C45	45	24420	6.3	67	56.7	**11.7**	**1.49**

Note: $f_{cu,k}=f_{cu,m}$ - 1.64 SD

For serviceability, accidental load and no-collapse checking, the **mean strength** of concrete is normally used. The following equations can yield more realistic and less conservative predictions.

$f_{cu,m} = f_{cu,d}$ **+ 9.8 + 10 MPa** C30 ≤ conc. Grade **(cube)** ≤ C45
or $f_{cu,m}$ = **1.5** $f_{cu,d}$; where 9.8 =1.64SD

$f_{c,m} = f_{c,d}$ **+ 8 + 8 MPa** C25 ≤ conc. Grade **(cylinder)** ≤ C35
or $f_{c,m}$ **= 1.5** $f_{c,d}$; where 8 =1.64SD

Concrete Compressive Strength

For seismic design, Priestley et al. (2007) recommended using the following expected mean concrete strength for checking of plastic hinge regions.

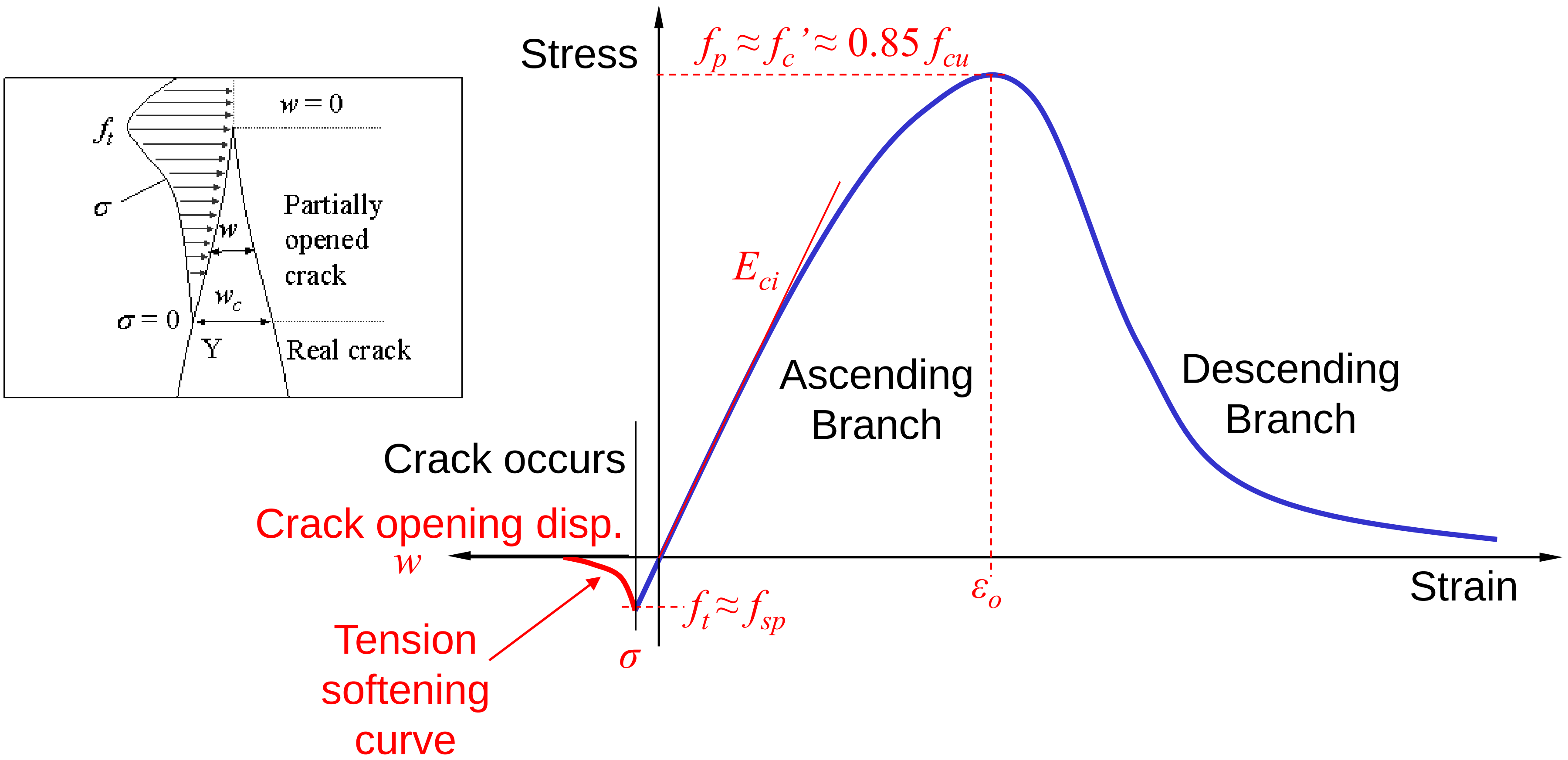
For lower-bound estimation	1.3 × characteristic strength
For upper-bound estimation	1.7 × characteristic strength
For local NSC C30 to C45	**1.5 × characteristic strength**

The proposed mean concrete strength is well within Priestley recommended lower- and upper-limits.

Certainly, if more concrete test data from various public and private projects is available, more reliable mean strength prediction formulas covering full range of concrete grades can be obtained.

Full Range Stress-Strain Curve

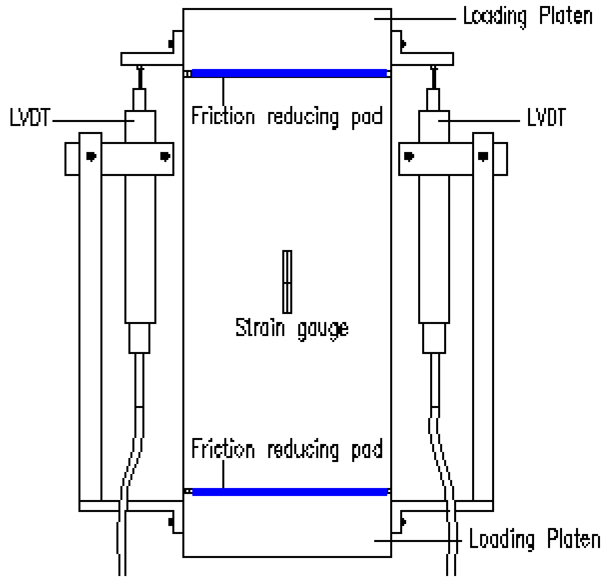
This curve is essential for non-linear analysis of concrete structures, such as the NLTHA for no-collapse seismic checking.



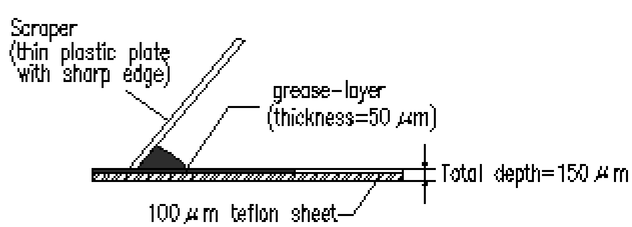
Full Range Stress-Strain Behaviour

Test Method

The standard testing method recommended by the RILEM committee (Shah et al., 2000) was adopted for determining the full-range stress-strain curve.



Test setup



Features:

A friction reducing pad was inserted between the specimen and loading platens.

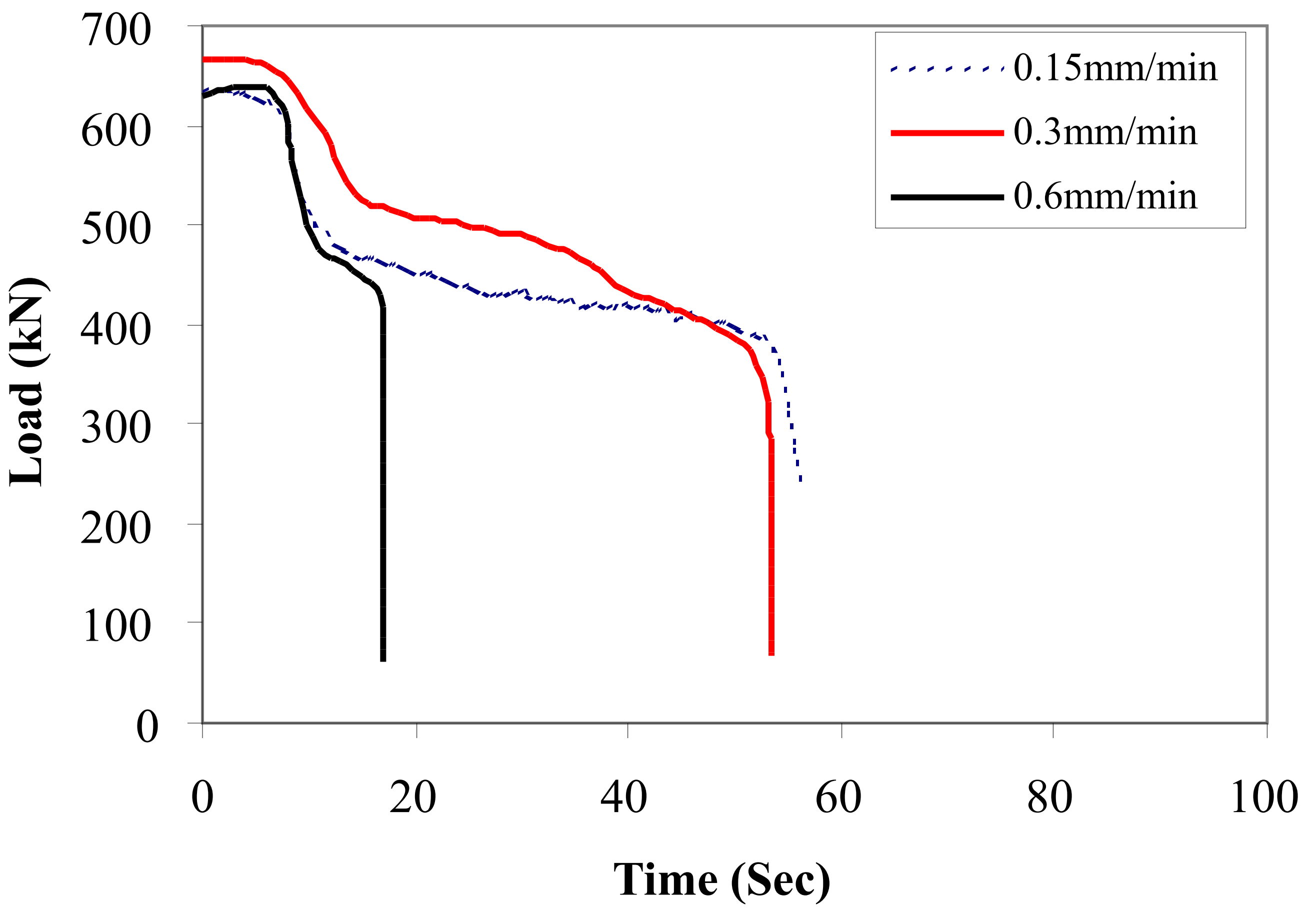
When the load increased to approx. 1/3 of the cube strength, force control is switched to displacement control at a rate of **1 μm/s**.

LVDTs are used to record the deformations.

E_{ci} is defined as the secant modulus at 40% of the peak strength.

Full Range Stress-Strain Behaviour

With the friction reducing pad , the loading drop (at post-peak stage) is dependent on loading rate. In this test, the loading rate of **1μm/s (or 0.06 mm/min)** was adopted.



Full Range Stress-Strain Behaviour

<u>Conventional Method</u>

Concrete cylinder without adding friction reducing pad between the specimen and loading platen

Failure Mode –
Split tension + shear failure

Conventional Method

No cracks on the loading faces of the cylinder after the test

Full Range Stress-Strain Behaviour

<u>RILEM Method</u>

Concrete cylinder with friction reducing pad between the specimen and loading platen

Failure Mode – mainly split tension failure

RILEM Method

Extensive vertical cracks and lateral movements occurred at the ends of the cylinder

Full Range Stress-Strain Behaviour

Mixed proportions of concrete prisms

Mix Series	Water (kg/m^3)	Cement (kg/m^3)	w/c	Fine Aggregates (kg/m^3)	Coarse Aggregates (kg/m^3)	Super Plasticizer (g/m^3)	Maximum Aggregates
3-10	200	279	0.72	1025	838	0	10
3-10	200	279	0.72	1025	838	0	20
4-10	193	332	0.58	905	905	432	10
4-20	193	332	0.58	905	905	432	20
6-10	196	423	0.45	867	866	3802	10
6-20	196	423	0.45	867	866	3802	20
8-10	173	482	0.35	867	866	6263	10
8-20	173	482	0.35	867	866	6263	20
9-10	160	501	0.32	867	866	8516	10
9-20	160	501	0.32	867	866	8516	20

10mm and 20 mm coarse aggregates were used and concrete cube strengths varied from 35 MPa to 90 MPa.

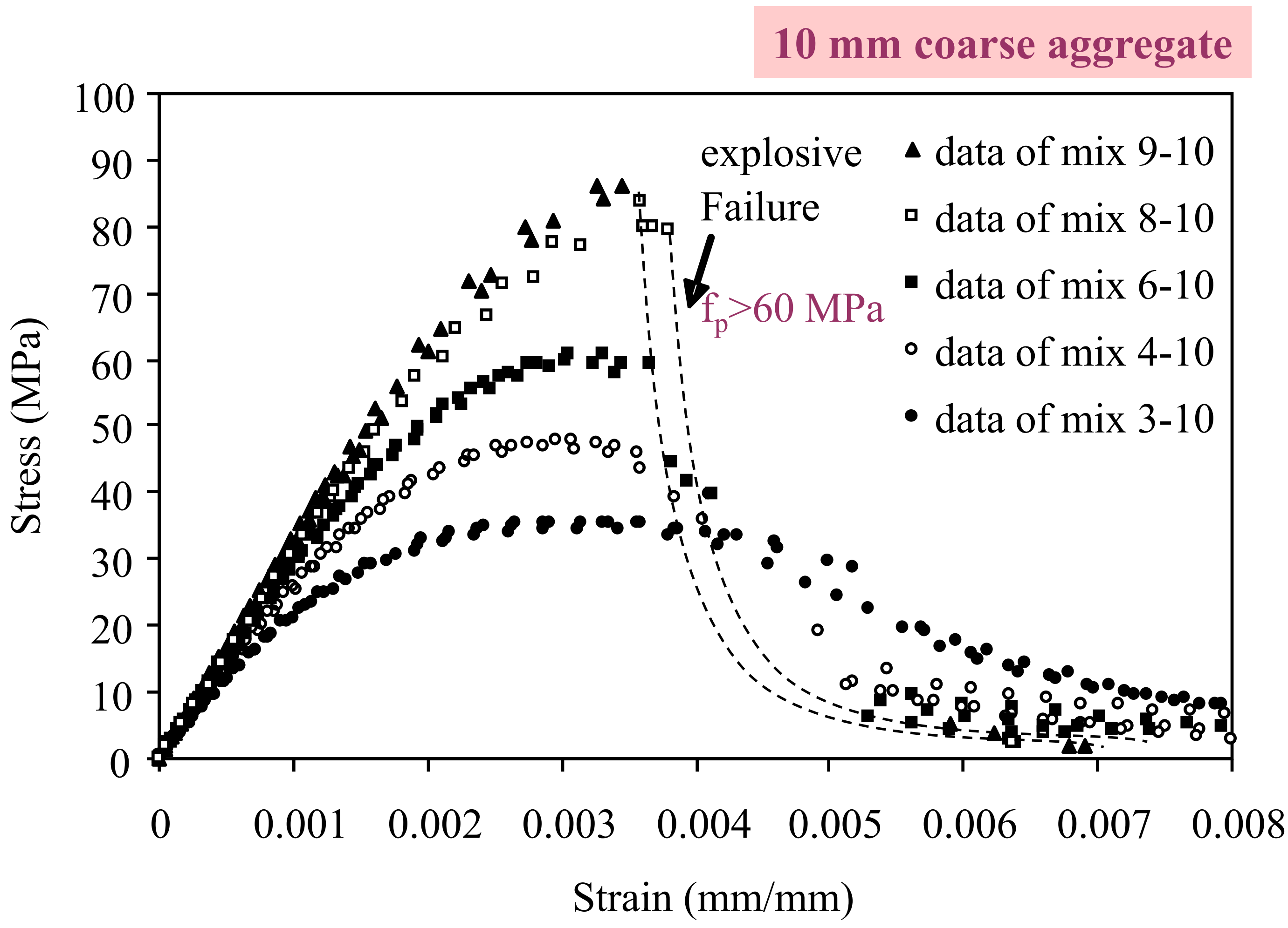
Effect of Max Aggregate Size on f_{cu}

Mix Series	The size of Maximum Coarse Aggregate	
	10mm	20mm
3-10 / 3-20	35.1 MPa	35.1 MPa
4-10 / 4-20	50.3 MPa	49.3 MPa
6-10 / 6-20	62.6 MPa	61.6 MPa
8-10 / 8-20	79.5 MPa	79.0 MPa
9-10 / 9-20	89.8 MPa	90.9 MPa

The size of maximum coarse aggregate in concrete does not have much effect on cube strength.

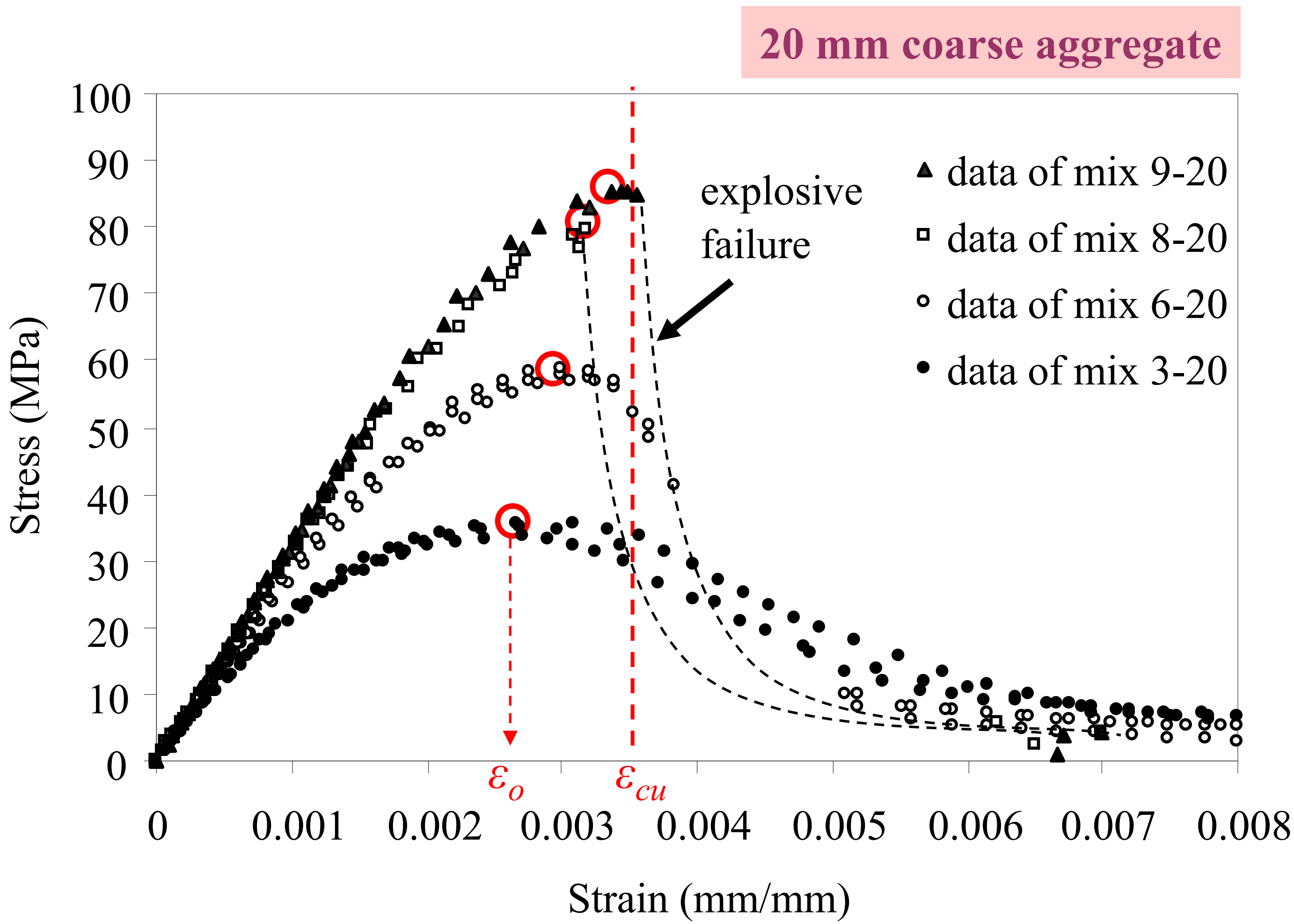
Full Range Stress-Strain Behaviour

The test results following RIELM method (Su and Cheng (2007))



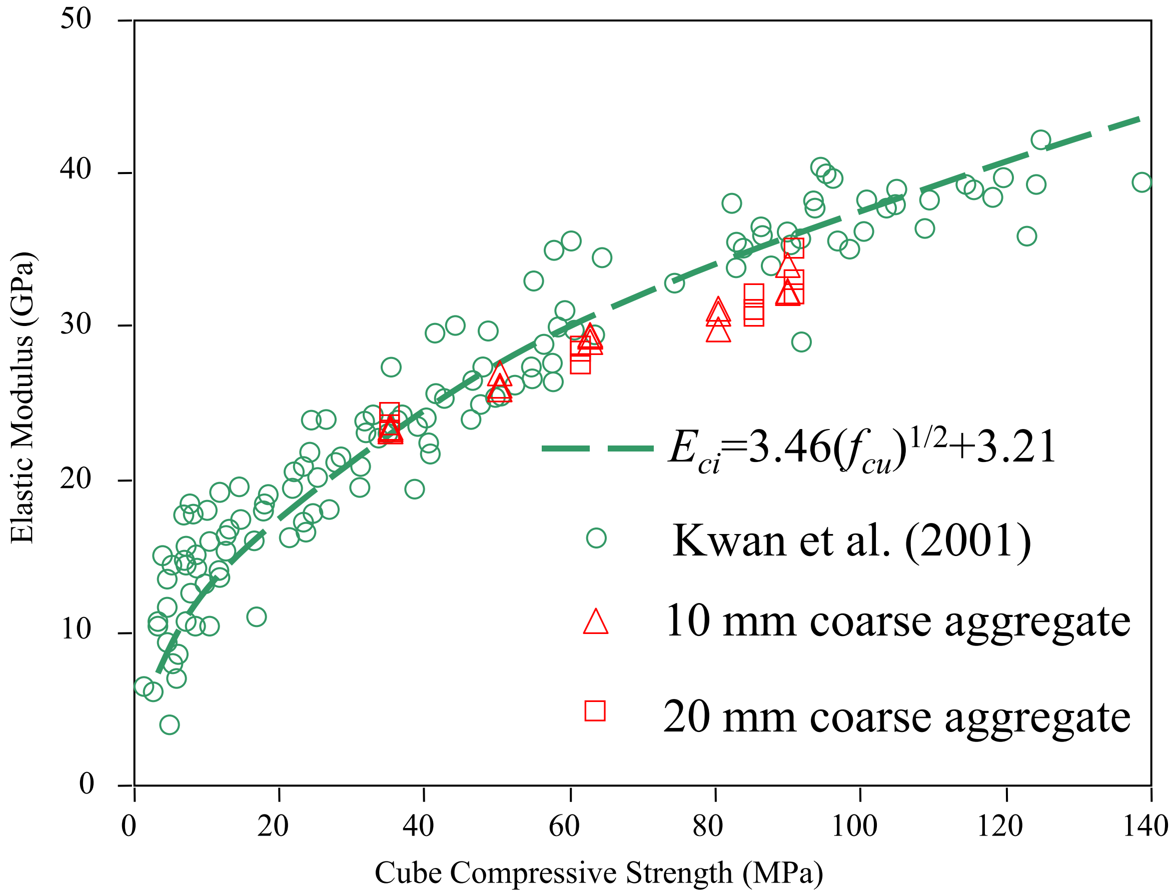
Full Range Stress-Strain Behaviour

The test results following RIELM method (Su and Cheng (2007))



Elastic Modulus

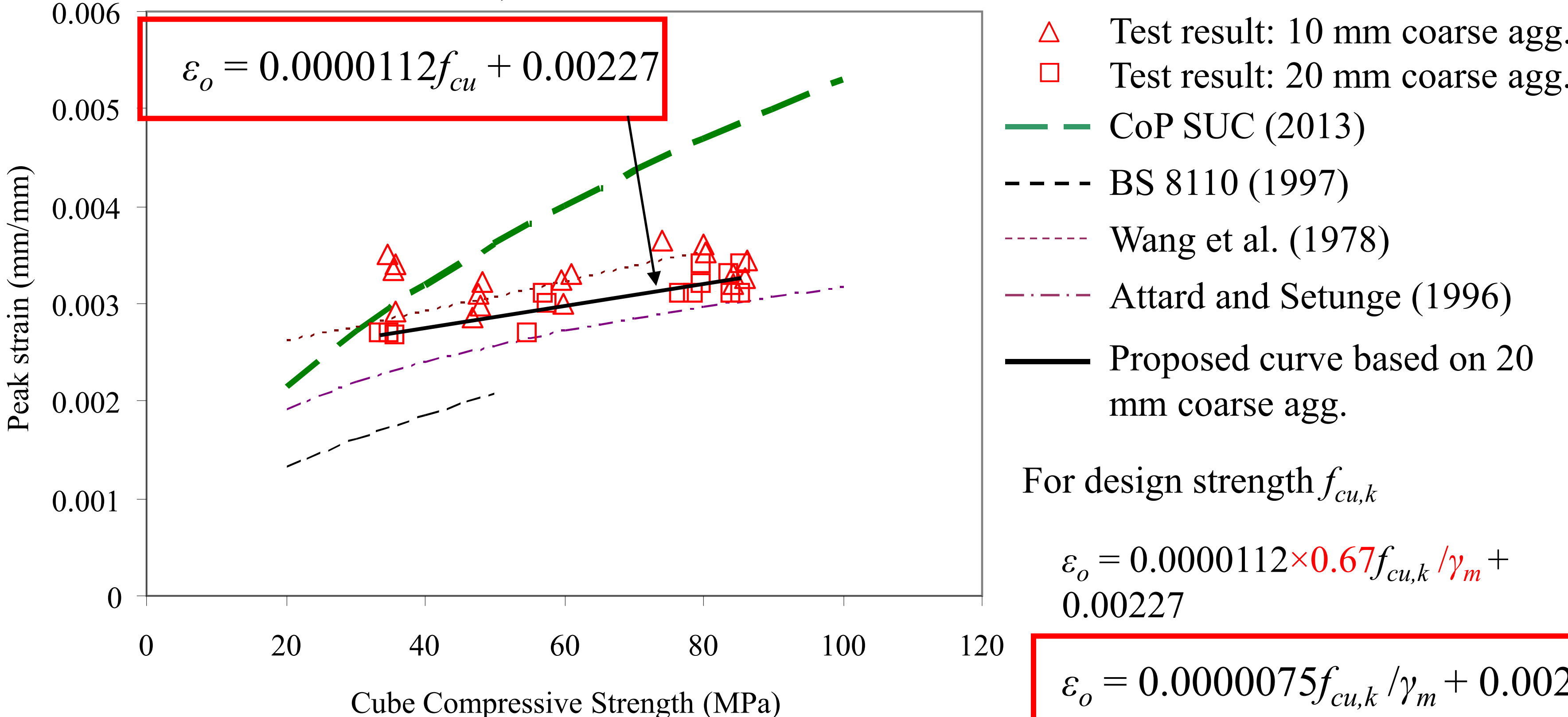
Kwan et al. (2001) determined the initial elastic modulus of local concrete according to BS1881(1983) and E_{ci} was defined as the secant modulus at 33% of concrete cube strength. Their results have been adopted in both CoP SUC (2013) and SDMH&R (2013).



The elastic modulus of concrete obtained from RIELM method, although is slightly lower than the codified values, is still within the normal scattering of the previous test results.

Proposed Peak Strain ε_o

For mean strength $f_{cu,m}$



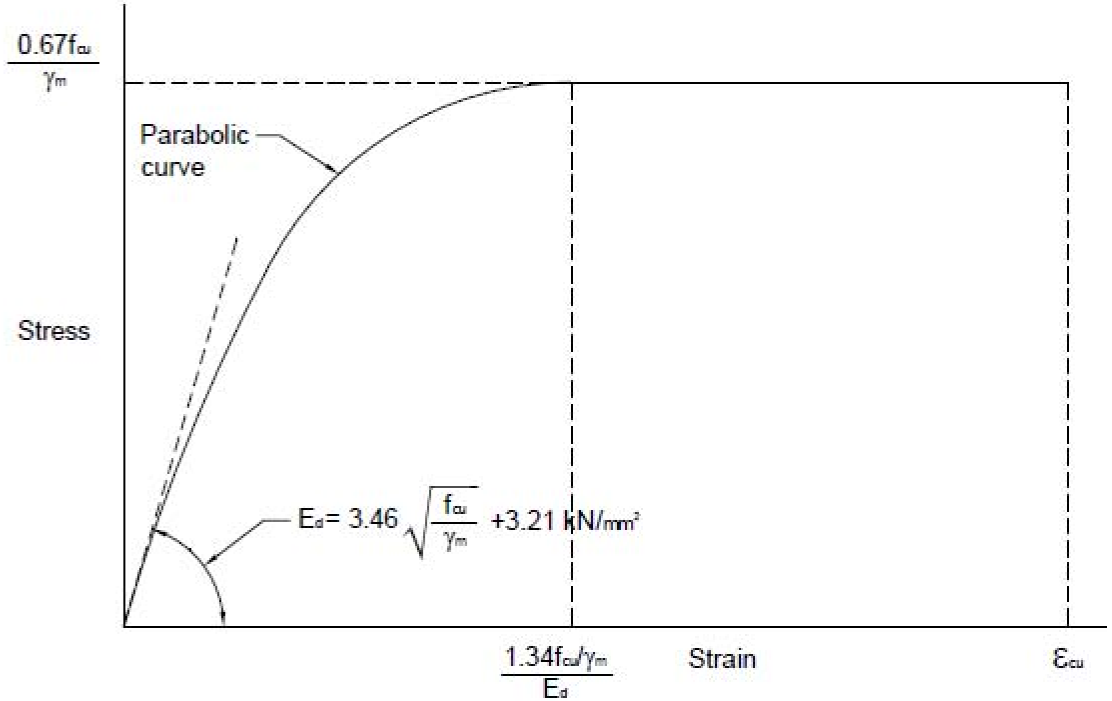
For design strength $f_{cu,k}$

$$\varepsilon_o = 0.0000112 \times 0.67 f_{cu,k} / \gamma_m + 0.00227$$

$$\varepsilon_o = 0.0000075 f_{cu,k} / \gamma_m + 0.00227$$

Codified Stress-Strain Curve

With material partial safety factor from CoP SUC (2013)

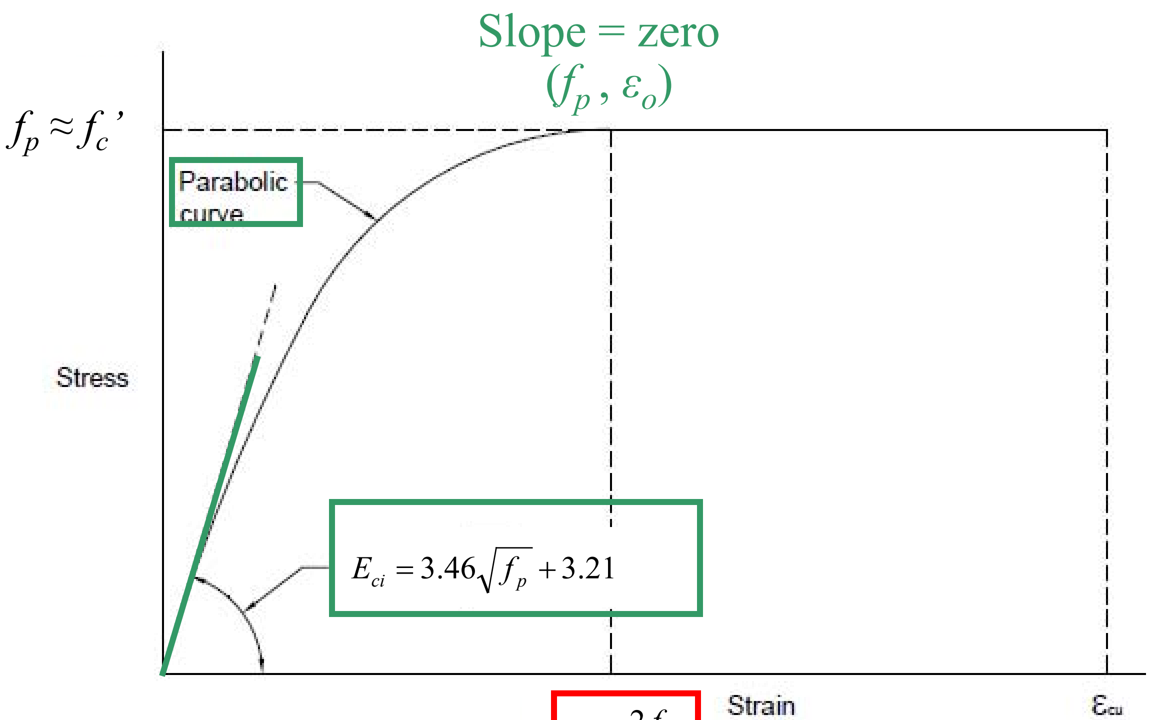


Notes :

1. 0.67 takes account of the relationship between the cube strength and the bending strength in a flexural member. It is simply a coefficient and not a partial safety factor.
2. f_{cu} is in N/mm^2
3. For $f_{cu} \leq 60$ MPa, $\varepsilon_{cu} = 0.0035$
   For $f_{cu} > 60$ MPa, $\varepsilon_{cu} = 0.0035 - 0.00006 \times \sqrt{(f_{cu} - 60)}$

Codified Stress-Strain Curve

Without material partial safety factor from CoP SUC (2013)



Peak strain

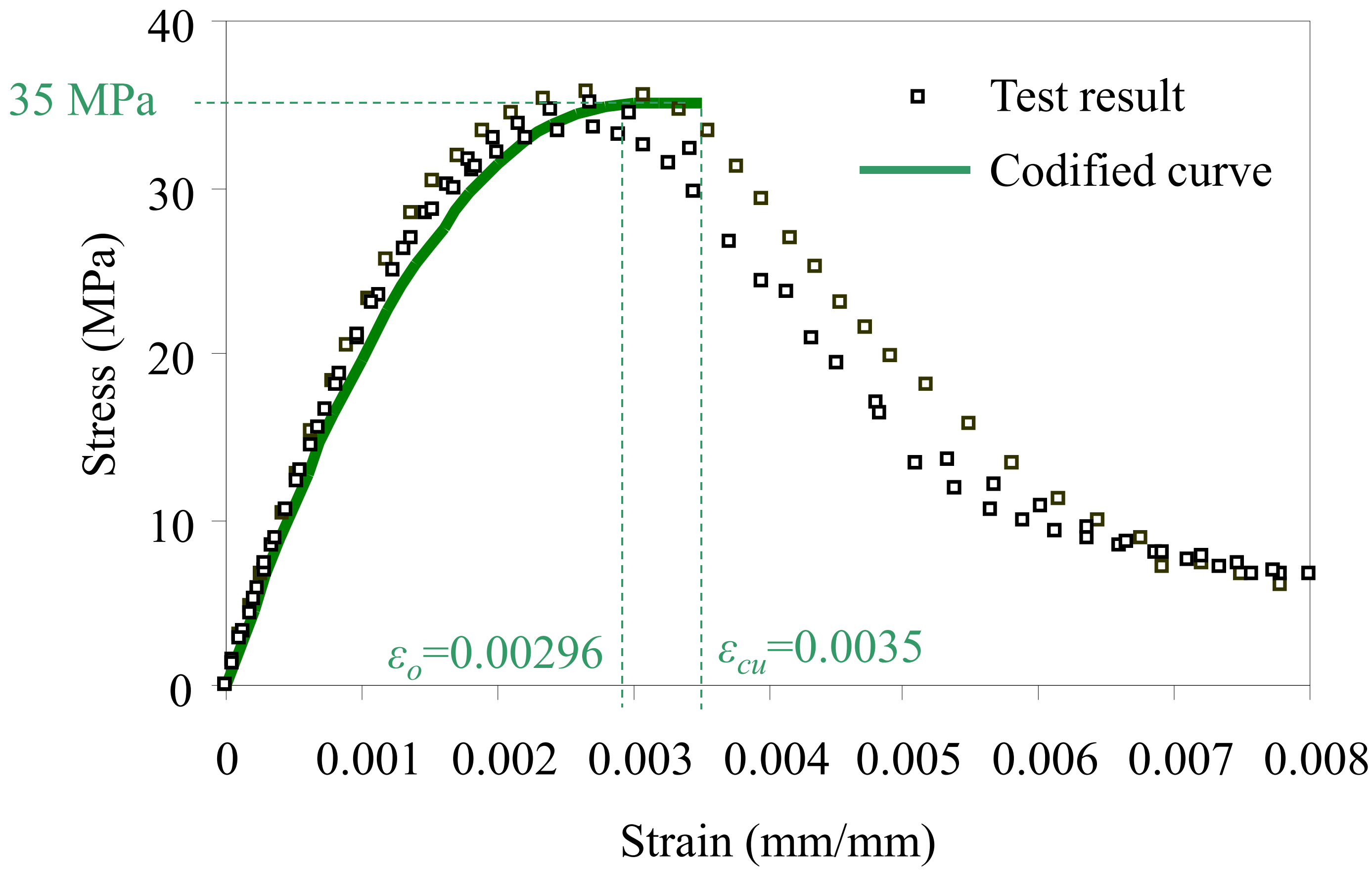
$$\varepsilon_o = \frac{2 f_p}{E_{ci}}$$

Parabolic equation

$$\sigma = E_{ci}\varepsilon + \left(\frac{f_p - E_{ci}\varepsilon_o}{\varepsilon_o^2}\right)\varepsilon^2$$

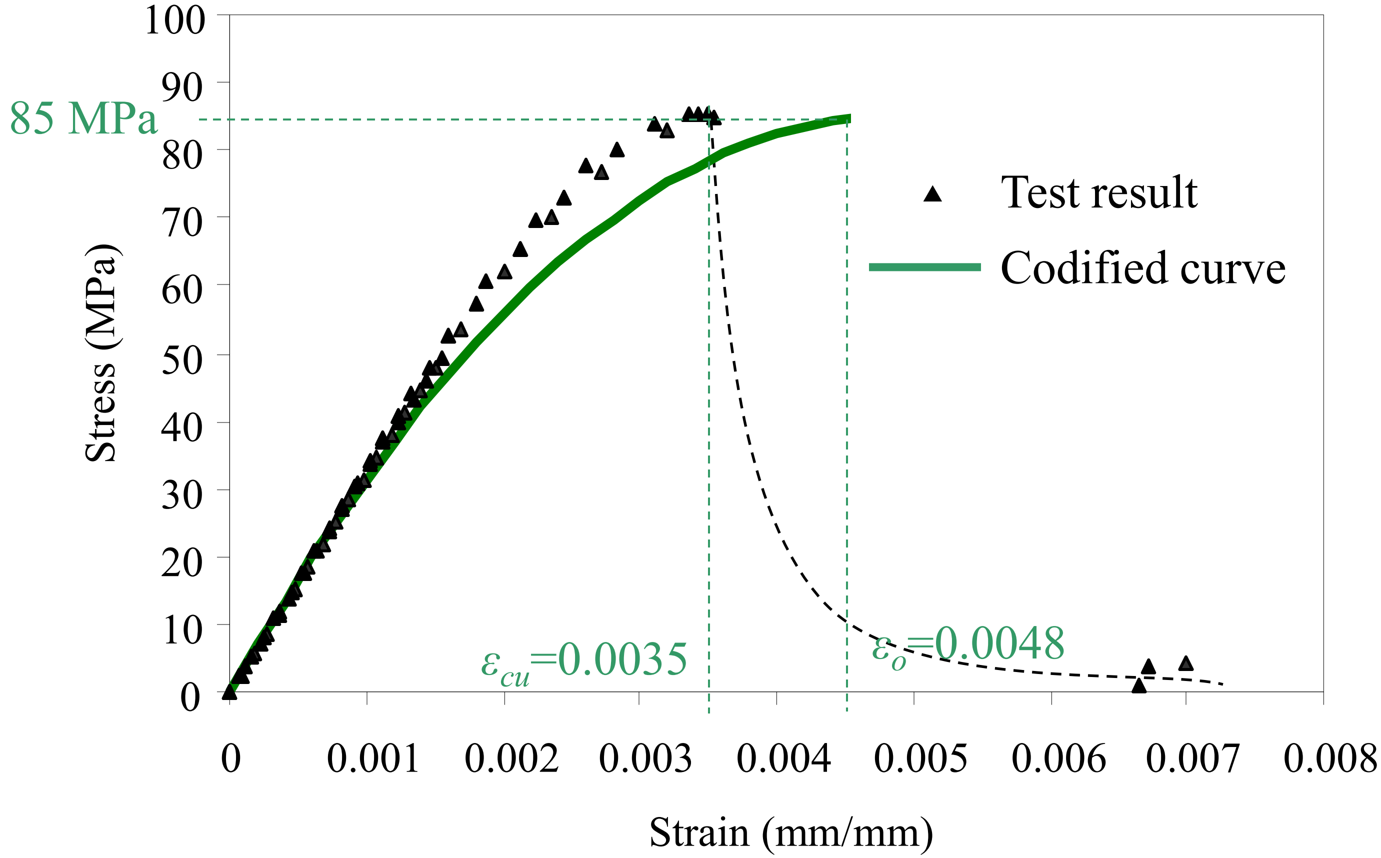
Comparison of the Stress-Strain Curve

Normal strength concrete f_p or $f_{c,m}$ = 35 MPa



Comparison of the Stress-Strain Curve

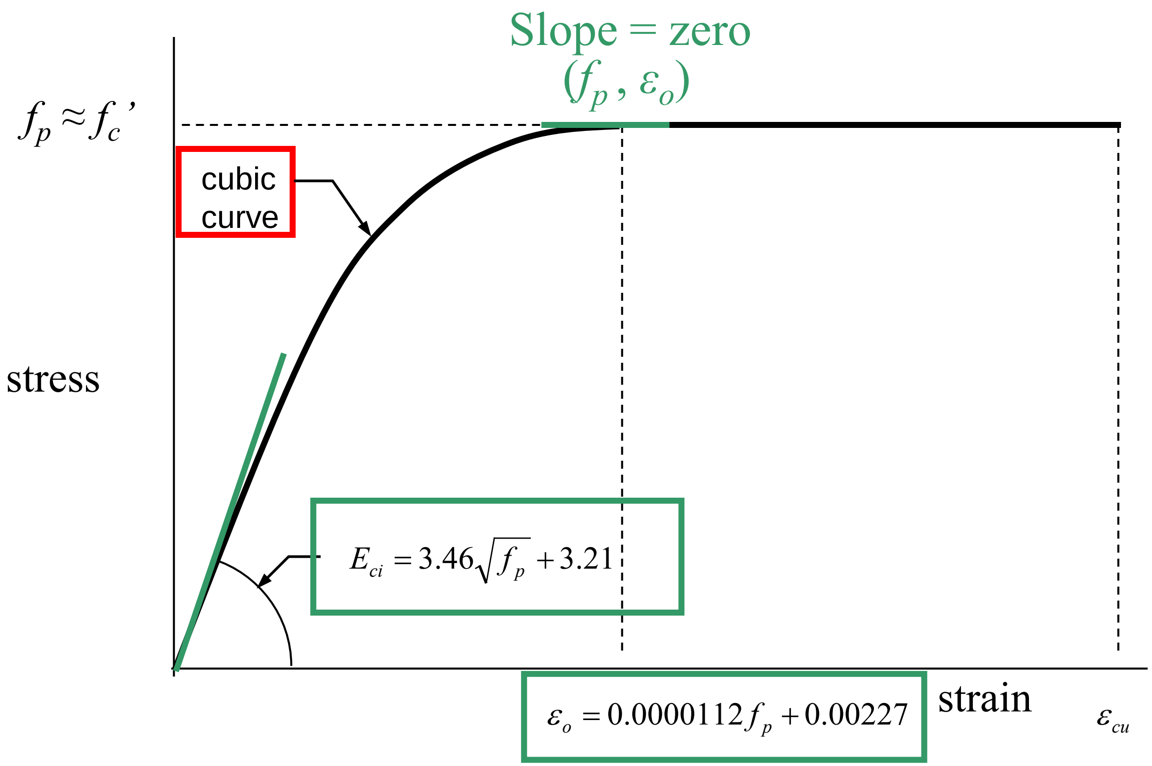
High strength concrete f_p or $f_{c,m}$ = 85 MPa



The predicted peak strain is much higher than the test results.

Proposed Stress-Strain Curve

Without material partial safety factor

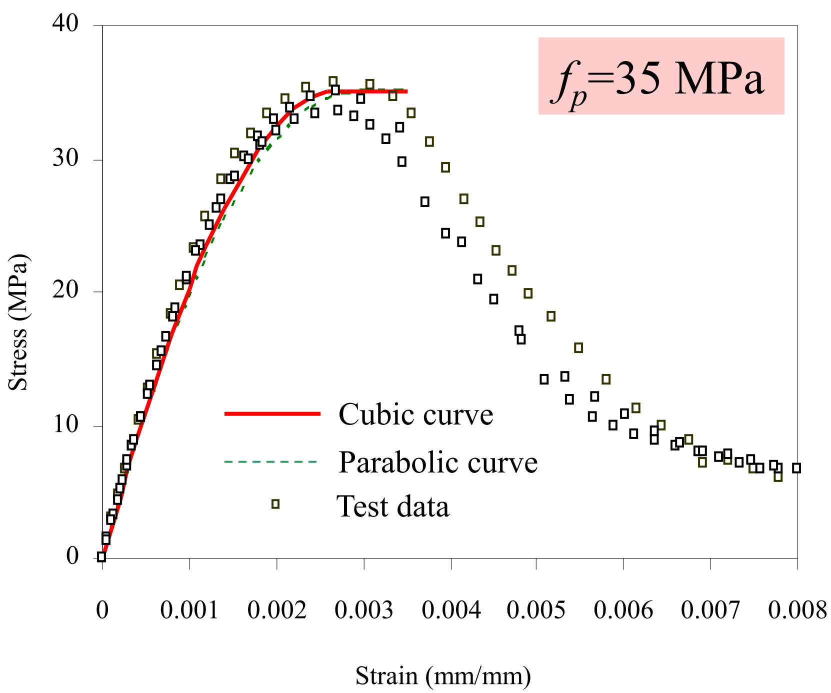


Cubic equation

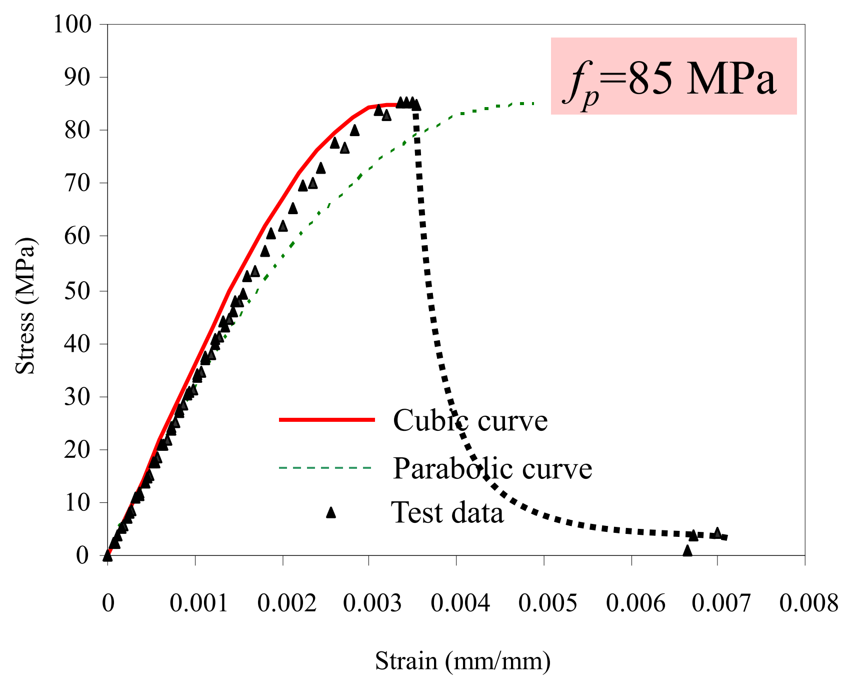
$$\sigma = E_{ci}\varepsilon + \left(\frac{3f_p - 2E_{ci}\varepsilon_o}{\varepsilon_o^2}\right)\varepsilon^2 + \left(\frac{E_{ci}\varepsilon_0 - 2f_p}{\varepsilon_o^3}\right)\varepsilon^3$$

Comparison of Stress-Strain Curves

Normal strength concrete



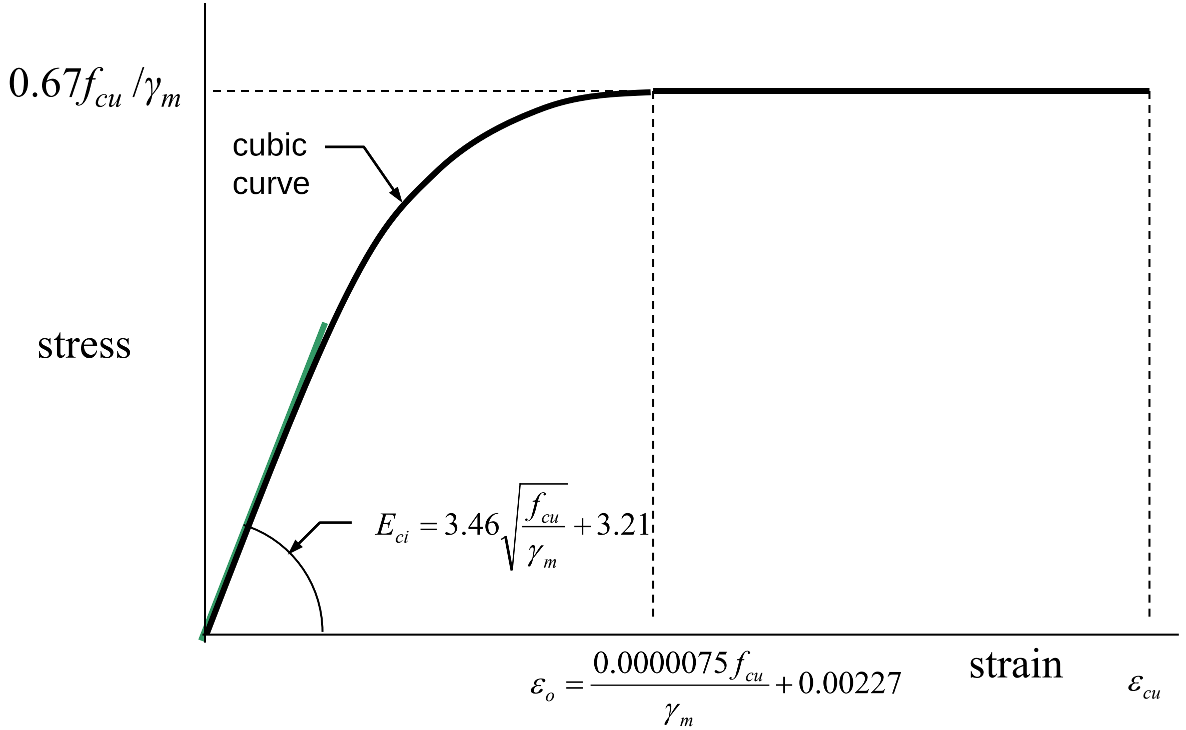
High strength concrete



The proposed cubic function is much more accurate than the current parabolic function.

Proposed Codified Stress-Strain Curve

With material partial safety factor



Proposed Full Range Stress-Strain Curve

Attard and Setunge (1996) stress-strain model

$$Y = \frac{AX + BX^2}{1 + CX + DX^2} \qquad Y = \frac{\sigma}{f_p}, X = \frac{\varepsilon}{\varepsilon_0}$$

The constants for the **ascending part** of the stress-strain curve:

$$A = \frac{E_{ci}\varepsilon_0}{f_p} \quad B = \frac{(A-1)^2}{0.55} - 1 \quad C = A - 2 \quad D = B + 1 \qquad (0 \le X, Y \le 1)$$

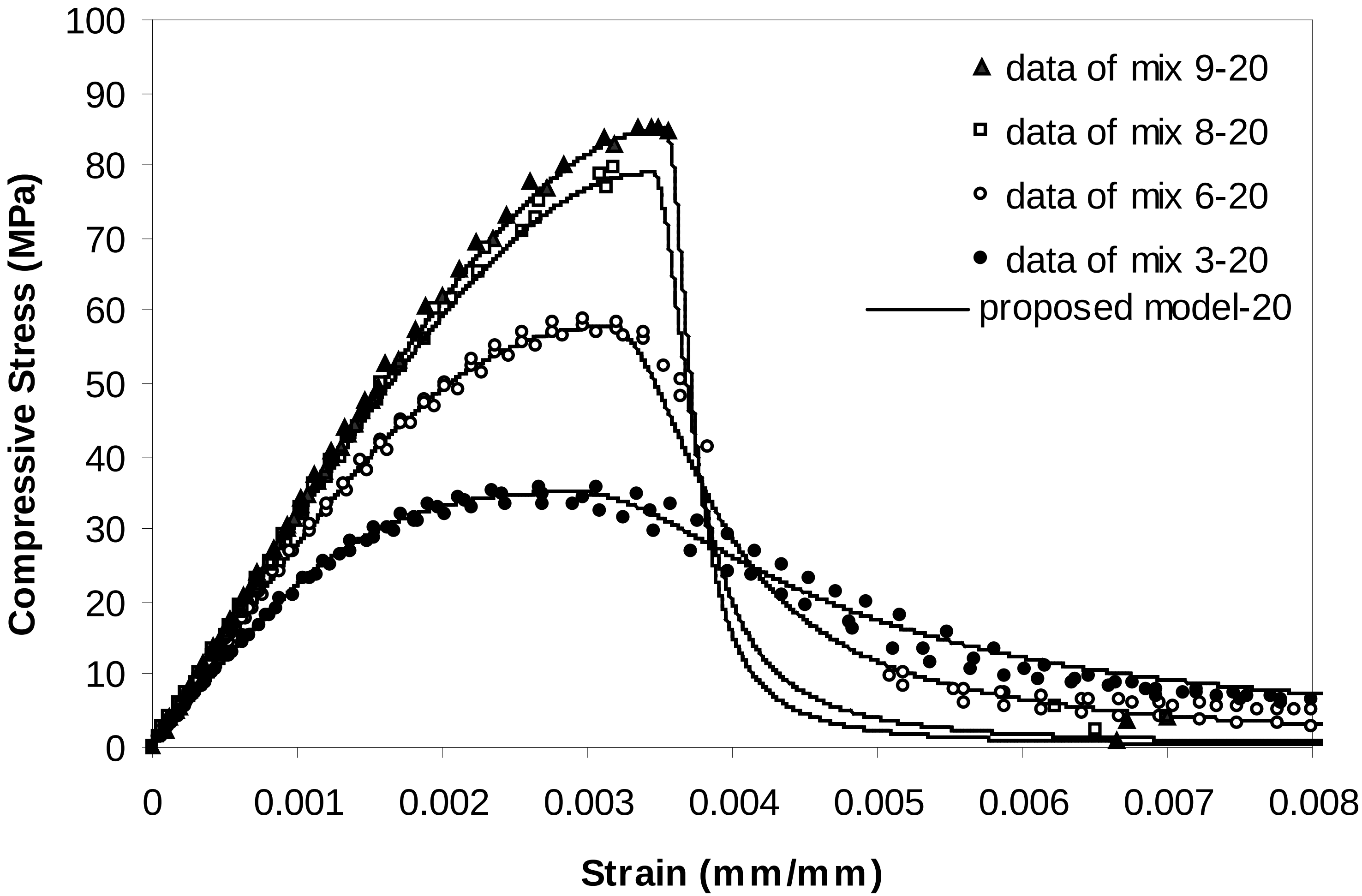
The constants for **descending part** of the stress-strain curve:

$$A = \frac{f_{ic}(\varepsilon_{ic} - \varepsilon_0)^2}{\varepsilon_0 \varepsilon_{ic}(f_p - f_{ic})} \quad B = 0 \quad C = A - 2 \quad D = 1 \quad (X > 1, Y > 1)$$

For concrete with 20 mm aggregates (Su and Cheng, 2008), the point of inflection is

$$\frac{\varepsilon_{ic}}{\varepsilon_o} = 6.2 - 1.12\ln\left(f_p\right) \quad \text{and} \quad \frac{f_{ic}}{f_p} = 1.25 - 0.26\ln\left(f_p\right)$$

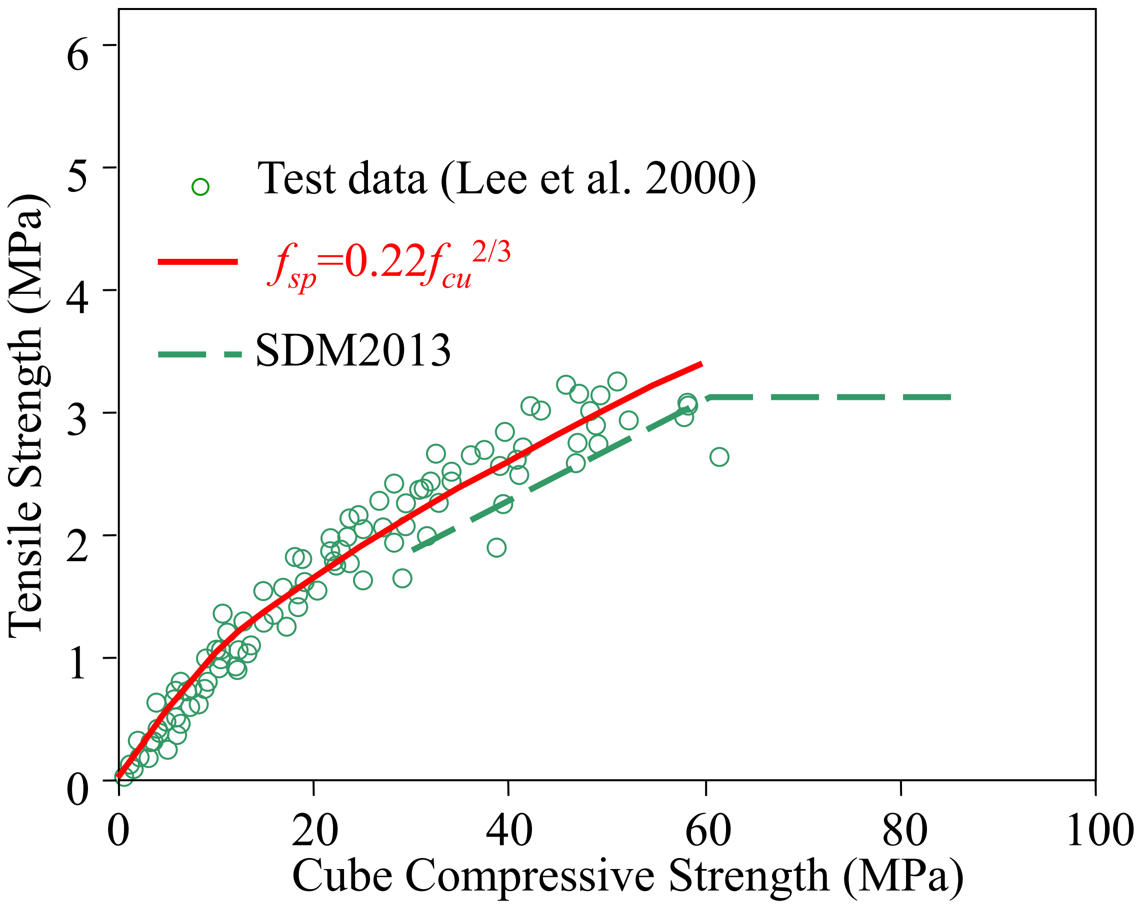
Proposed Full Range Stress-Strain Curve



The proposed full range stress-strain model is validated by the experimental data.

Tensile Strength

Lee et al. (2000) conducted split cylinder tests to investigate the tensile strength of local normal strength concrete with cube compressive strengths varying from 1 to 60 MPa. The tests were conducted according to the relevant parts of the British Standards and the Hong Kong Construction Standards.



Tensile Strength

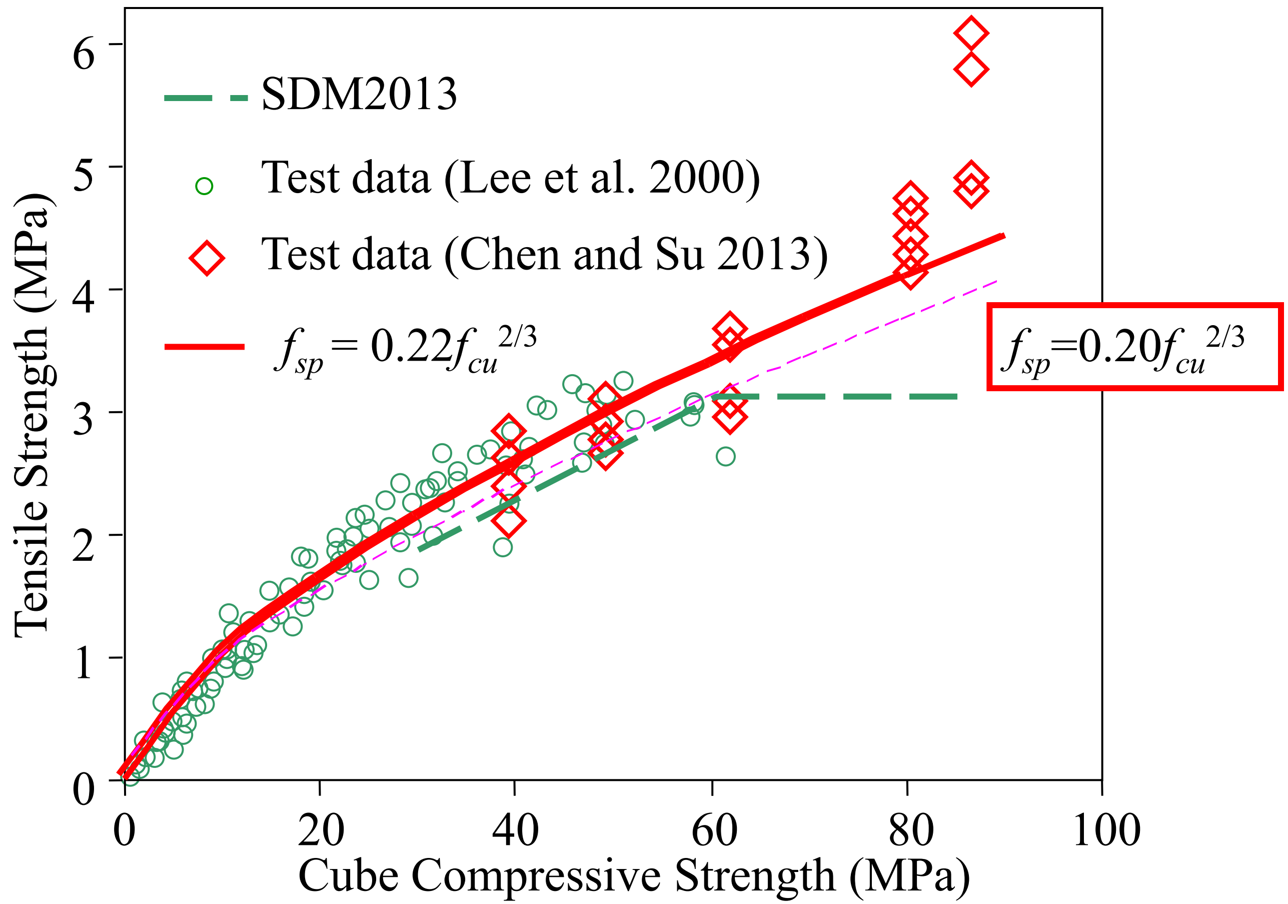
Chen and Su (2013) conducted split cylinder tests to obtain the tensile strength of local concrete with cube strengths varying from 40 to 87 MPa.

- Mix proportions and material properties

Specimen series	Water (kg/m^3)	Cement (kg/m^3)	w/c	Fine aggregate (kg/m^3)	Coarse aggregate (kg/m^3)	Super plasticiser (g/m^3)	f_{cu} (MPa)	f_{sp} (MPa)
C40	200	279	0.72	1025	838	0	39.7	2.5
C50	193	332	0.58	905	905	432	49.1	2.9
C60	196	423	0.46	867	866	3802	62.2	3.3
C80	173	482	0.36	867	866	6263	80.5	4.5
C90	160	501	0.32	867	866	8516	86.7	5.4

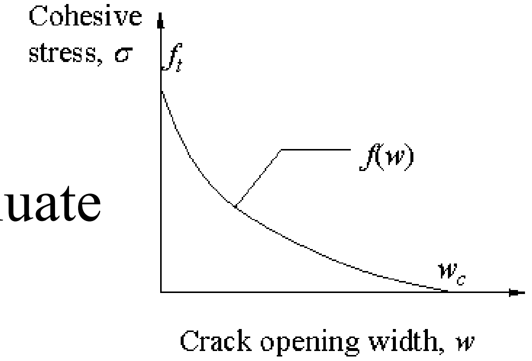
$$f_{sp} = \frac{2P}{\pi HD}$$

Proposed Tensile Strength Relationship

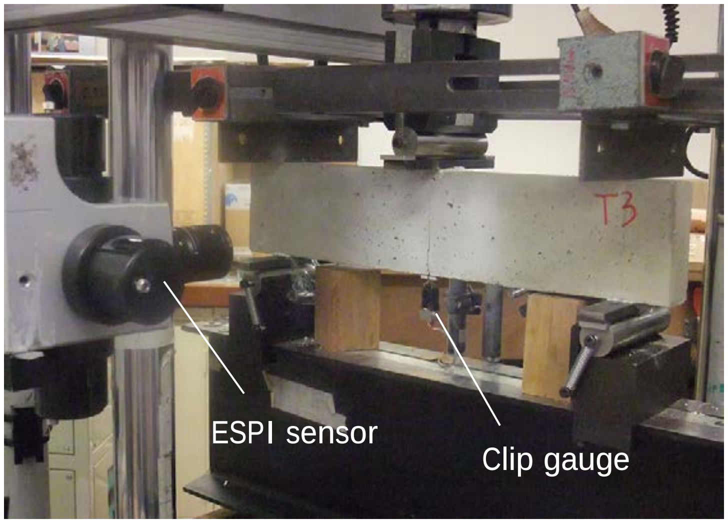


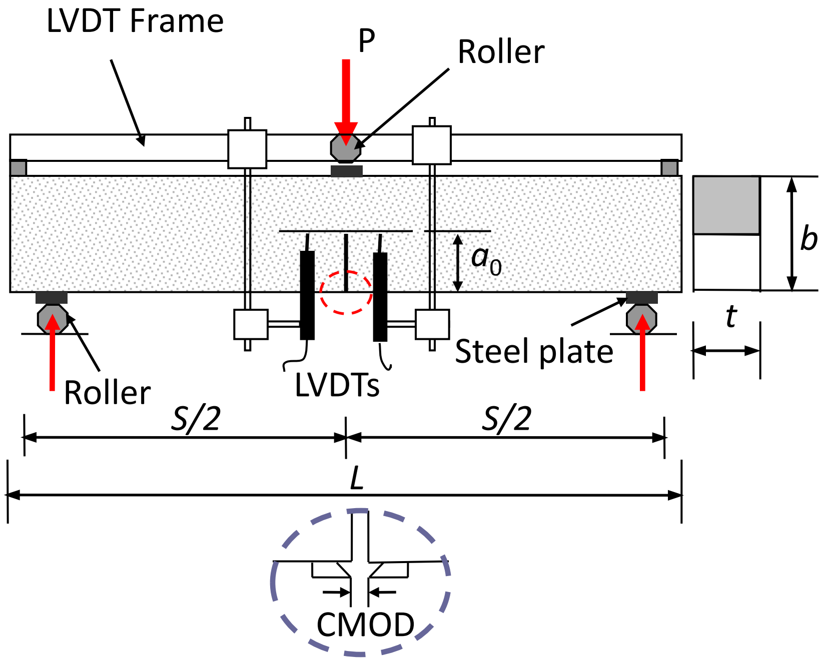
Tension Softening Curve

Chen and Su (2013) developed the Incremental Displacement Collocation Method (IDCM) to evaluate the TSC of concrete.



Experimental Test





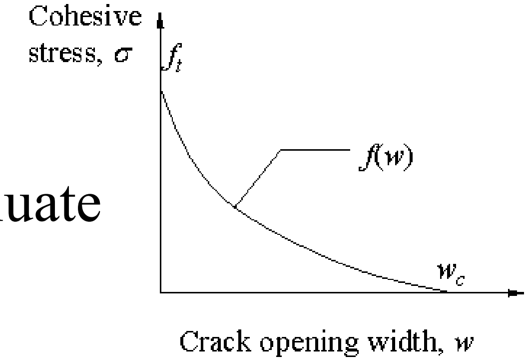
	L	S	b	t	a_0
mortar	500	400	100	40	30
concrete	710	600	150	80	45

Unit: mm

- Loading rate:0.01mm/min
- Gypsum packing, steel plate at the loading points
- Additional frame used as LVDT reference

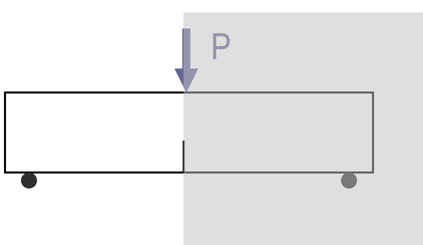
Tension Softening Curve

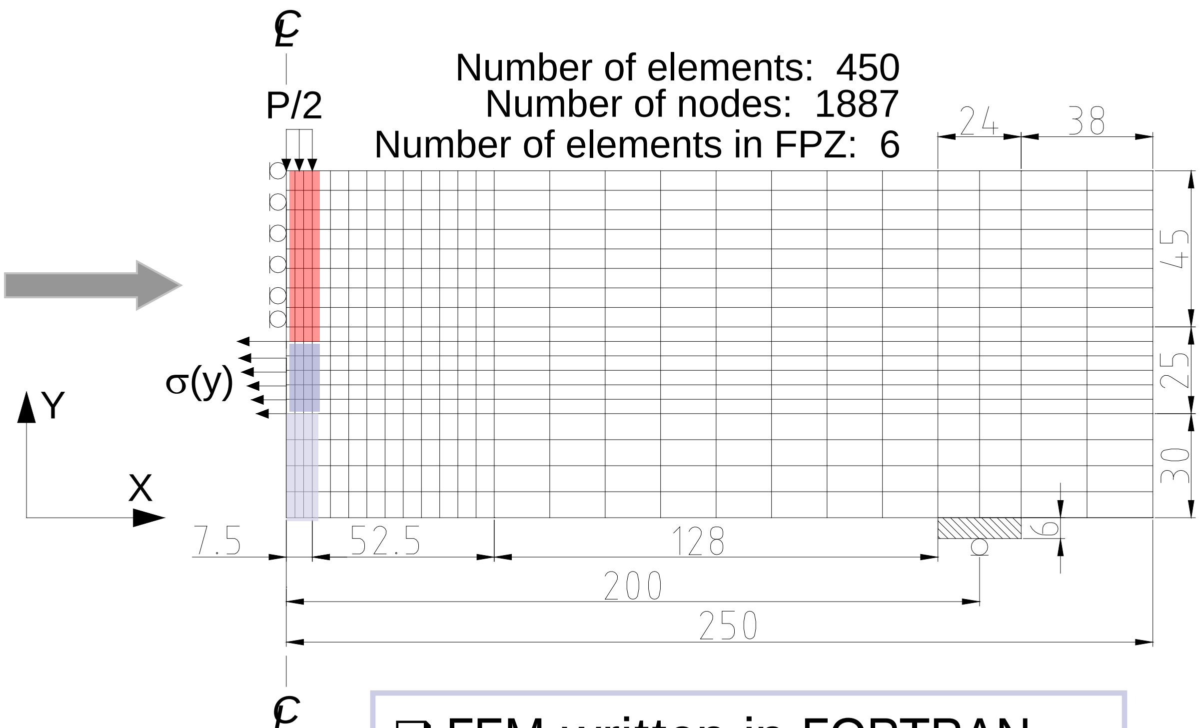
Chen and Su (2013) developed the Incremental Displacement Collocation Method (IDCM) to evaluate the TSC of concrete.



Numerical Model

The FEM





Linear elastic zone

Cohesive crack zone

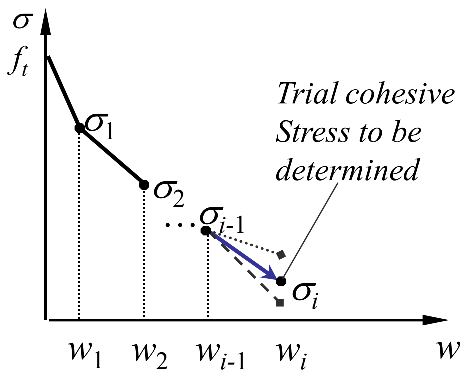
Real crack zone

- ❑ FEM written in FORTRAN
- ❑ 9-node hybrid element

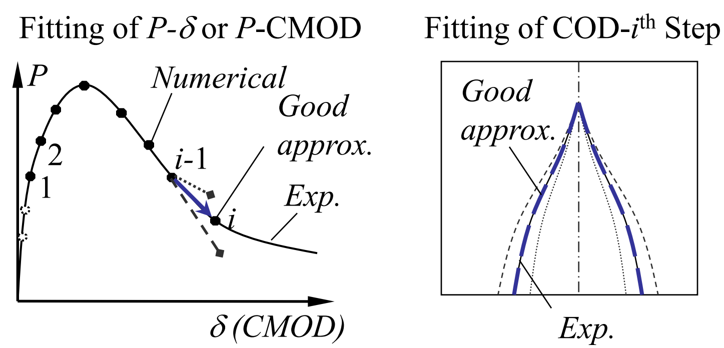
Tension Softening Curve

Implementation of the IDCM

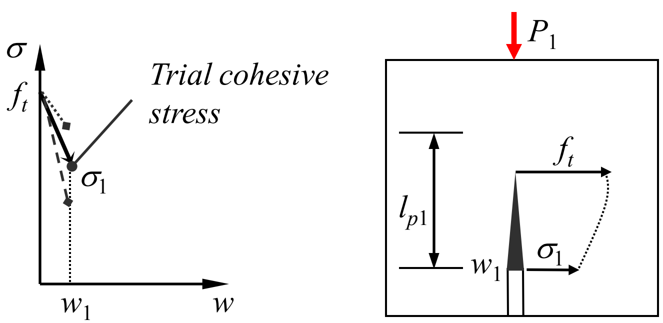
Incremental construction of the TSC



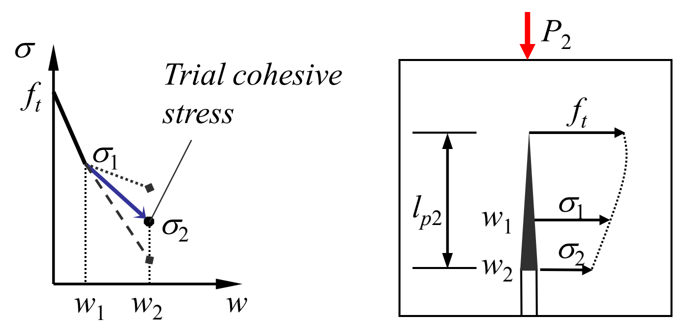
Displacement collocation process



Estimation of the 1st trial cohesive stress

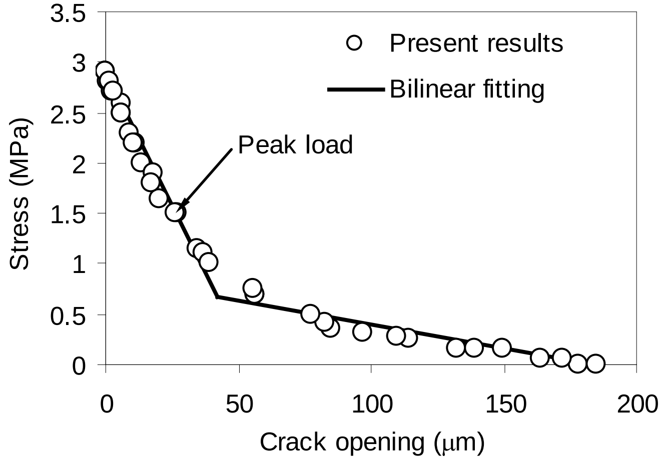


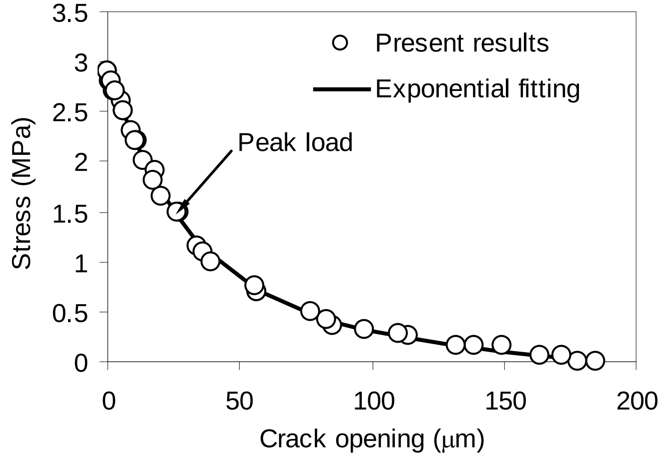
Estimation of the 2nd trial cohesive stress



Tension Softening Curve

Key parameters for evaluating the TSC





Series	Fracture parameters			Bilinear curve					Exponential curve			
	G_F (N/m)	w_c (μm)	f_t (MPa)	w_1 (μm)	f_1 (MPa)	w_1/w_c	f_1/f_t	RMSD	c_1	c_2	c_3	RMSD
C40	96.4	214.0	2.5	22.9	0.7	0.11	0.28	0.075	1.5	6.0	5.6	0.201
C50	120.1	184.3	2.9	42.0	0.7	0.23	0.23	0.089	1.8	6.5	5.8	0.047
C60	129.6	238.3	3.3	33.0	0.7	0.14	0.21	0.137	2.9	7.9	6.1	0.108
C80	124.2	188.7	4.5	20.3	1.0	0.11	0.22	0.262	5.0	11	6.8	0.146
C90	114.6	165.0	5.4	12.0	1.2	0.07	0.22	0.258	6.0	13	7.2	0.187

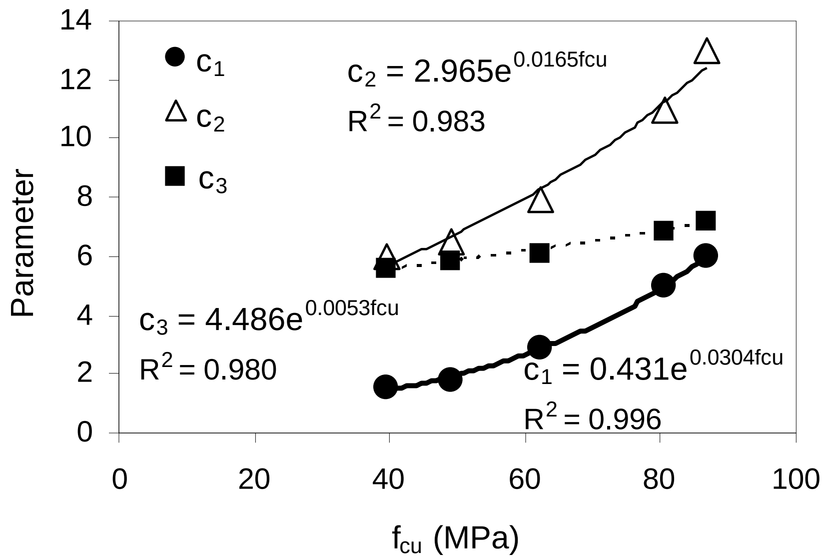
Tension Softening Curve

Key parameters for evaluating the TSC

f_{cu} varying from 39.7 MPa to 89.7 MPa

$$\frac{\sigma}{f_t} = \left\{1 + \left(c_1 \frac{w}{w_c}\right)^3\right\}\exp\left(-c_2 \frac{w}{w_c}\right) - \frac{w}{w_c}\left(1 + c_1^{\ 3}\right)\exp(-c_2)$$

and $$w_c = c_3 \frac{G_F}{f_t}$$



$$c_1 = 0.431e^{0.0304 f_{cu}}$$

$$c_2 = 2.965e^{0.0165 f_{cu}}$$

$$c_3 = 4.486e^{0.0053 f_{cu}}$$

Max Shear Strength Limit

Max shear strength is useful for the design of shear controlled elements, e.g. corbels, transfer beams, pile caps, etc.
Various empirical models have been adopted in different concrete codes for determining the maximum shear strength limit of concrete

- BS 8110 method

$$v_c = 0.79(\rho)^{1/3}(400/d)^{1/4}(f_{cu}/25)^{1/3}/\gamma_{shear} \quad \le \text{lesser of } \left[0.8\sqrt{f_{cu}},\ 5\,\text{MPa}\right]$$

- HK CoP Concrete code 2013 method

$$v_c = 0.79(\rho)^{1/3}(400/d)^{1/4}(f_{cu}/25)^{1/3}/\gamma_{shear} \quad \le \text{lesser of } \left[0.8\sqrt{f_{cu}},\ 7\,\text{MPa}\right]$$

- GB 50010 method

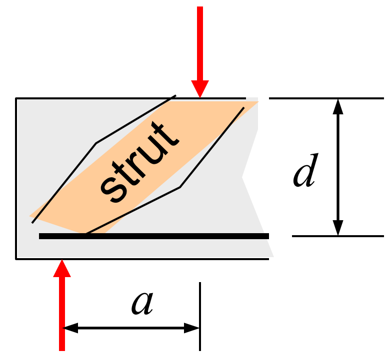
$$v_c = 0.7(0.7+20\rho)(800/d)^{1/4} f_{tk}/\gamma_{shear} \quad \le 0.20\beta_c f_c'$$

- ACI 318-11 method

$$v_c = 0.16\lambda\sqrt{f_c'} + 17\rho V_u/M_u d \quad \le 0.29\lambda\sqrt{f_c'}$$

- EC2 method

$$v_c = C_{Rd,ck}(\rho f_{ck})^{1/3} + k_1\sigma_{cp} \quad \le 0.5\beta f_{cd}$$



Max shear strength of concrete is limited by the axial capacity of the diagonal strut

Which equation is more rational?

Max Shear Strength Limit

Experimental studies on shear capacity of unreinforced webs

Reference	No.	fc' (M Pa)	d (mm)	a/d	STM angle
Moody et al. (1954)	11	41.1 to 58.9	533	1.5	33.3
Mathey & Watstein (1963)	16	21.9 to 27.0	403	1.5	33.5
Leonhardt & Walter (1964)	1	21.0	270	1.10	42.3
de Pavia & Siess (1965)	2	19.9 to 23.3	203	1.0	45.0
Kani (1967)	17	24.8 to 31.5	132 to 1097	1.0 to 2.5	21.8 to 45
Ramakarishnan et al. (1968)	2	11.2 to 13.4	349	0.5	63.9
Placas (1969)	3	34.0	272	1.99	26.7
Crist (1970)	1	22.5	914	0.8	53.1
Manuel et al. (1971)	4	30.1 to 35.2	406	0.3	73.3
Smith & Vantsiotis (1982)	2	20.5 to 20.9	305	1.0	45.0
Smith & Vantsiotis (1982)	2	19.5 to 20.7	305	1.34 to 2.01	26.4 to 36.7
Rogowsky et al. (1983)	10	26.1	950	1.0	45.0
Rogowsky et al. (1986)	7	42.4 to 43.2	455 to 535	1.87 to 2.2	22.4 to 28.1
Lehwalter (1988)	4	14.0 to 19.0	200 to 1000	1.25	38.7
Reyes de Ortiz (1993)	5	32.0 to 51.0	326 to 363	1.10 to 1.38	35.9 to 42.3
Fang et al. (1995)	4	29.8 to 85.9	500	0.5 to 1.0	45.0 to 63.4
Papadakis (1996)	8	24.0 to 25.3	200	1.5 to 2.0	26.5 to 33.7
Tan et al. (1997)	1	78.0	443	1.41	35.3
Vollum RL & Tay UL (2001)	10	25.0 to 44.0	180	1.14 to 1.28	38.0 to 41.3
Cheng et al. (2001)	4	39.0 to 44.0	444 to 1559	1.46 to 1.56	32.7 to 34.4
KH Yang et al. (2003)	21	31.4 to 78.5	355 to 935	0.53 to 1.1	42.3 to 63.4
Quintero-Febres et al. (2006)	6	22.0 to 50.3	370 to 380	0.81 to 1.43	25.0 to 43.8
Zhang & Tan (2007)	8	24.8 to 32.4	314 to 926	1.1	42.3
Sagaseta (2008)	2	68.0 to 80.0	438	1.12	41.8
Sahoo et al. (2009)	2	39.6 to 45.2	360	0.5	63.4
Database without web rebars	**153**	**11.2 to 85.9**	**132 to 1559**	**0.3 to 2.5**	**21.8 to 73.3**

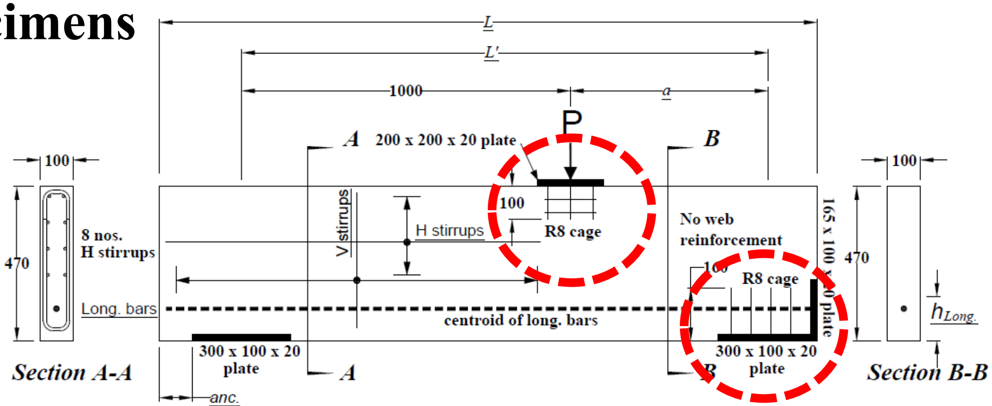
Limited data for high strength concrete f_c' > 60 MPa

Beam Tests

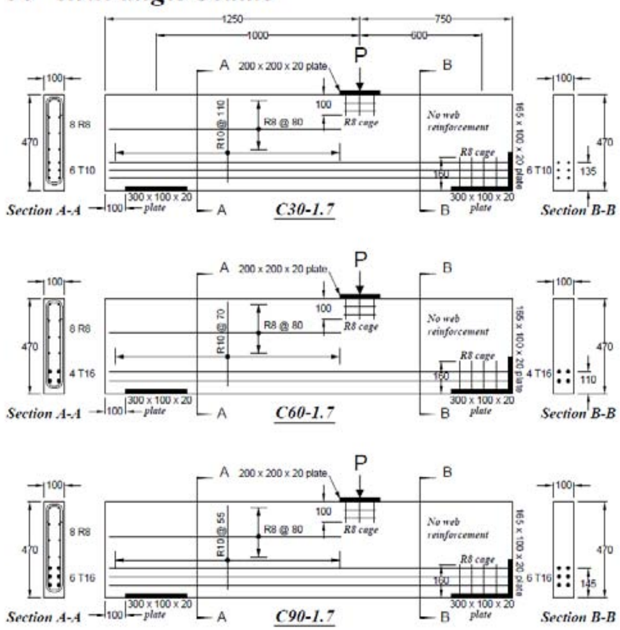
Cage to protect NODES at loading point and support

Details of specimens

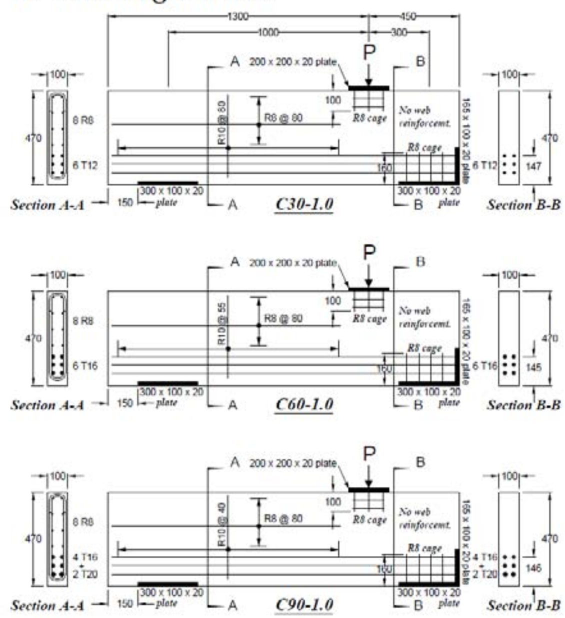
f_{cu} ranging from 34 MPa to 97 MPa



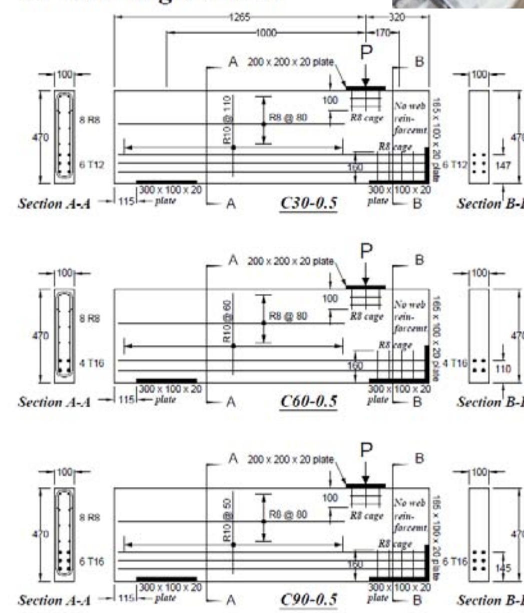
30° strut angle beams



45° strut angle beams



60° strut angle beams

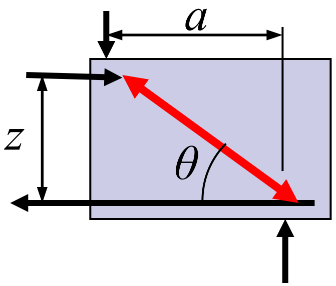


$a/z = 1.7$ $a/z = 1.0$ $a/z = 0.5$

Beam Tests

Details of the specimens

Type	L mm	L' mm	a mm	$anc.$ mm	H stirrups	V stirrups	Long. bars.	ρ %	$h_{Long.}$ mm	z mm	a/z	θ °	f_{cu} MPa
C30-1.7	2000	1600	600	100	R8@80	R10@110	6 T10	1.00	135	336.5	1.78	29.3	34.1
C60-1.7						R10@70	4 T16	1.71	110	330.5	1.82	28.8	64.7
C90-1.7						R10@55	6 T16	2.57	145	308.1	1.95	27.2	89.5
C30-1.0	1750	1300	300	150		R10@80	6 T12	1.44	147	339.2	0.88	48.5	34.8
C60-1.0						R10@55	6 T16	2.57	145	330.7	0.91	47.8	66.1
C90-1.0						R10@40	4 T16 2 T20	3.05	146	317.1	0.95	46.6	97.0
C30-0.5	1585	1170	170	115		R10@110	6 T12	1.44	147	341.5	0.50	63.5	33.9
C60-0.5						R10@60	4 T16	1.71	110	336.7	0.50	63.2	65.3
C90-0.5						R10@50	6 T16	2.57	145	353.8	0.48	64.3	92.6



All specimens were designed to strut failure.

Max Shear Strength Limit

Normalised measured concrete shear stress v_c with f_{cu} or $\sqrt{f_{cu}}$?

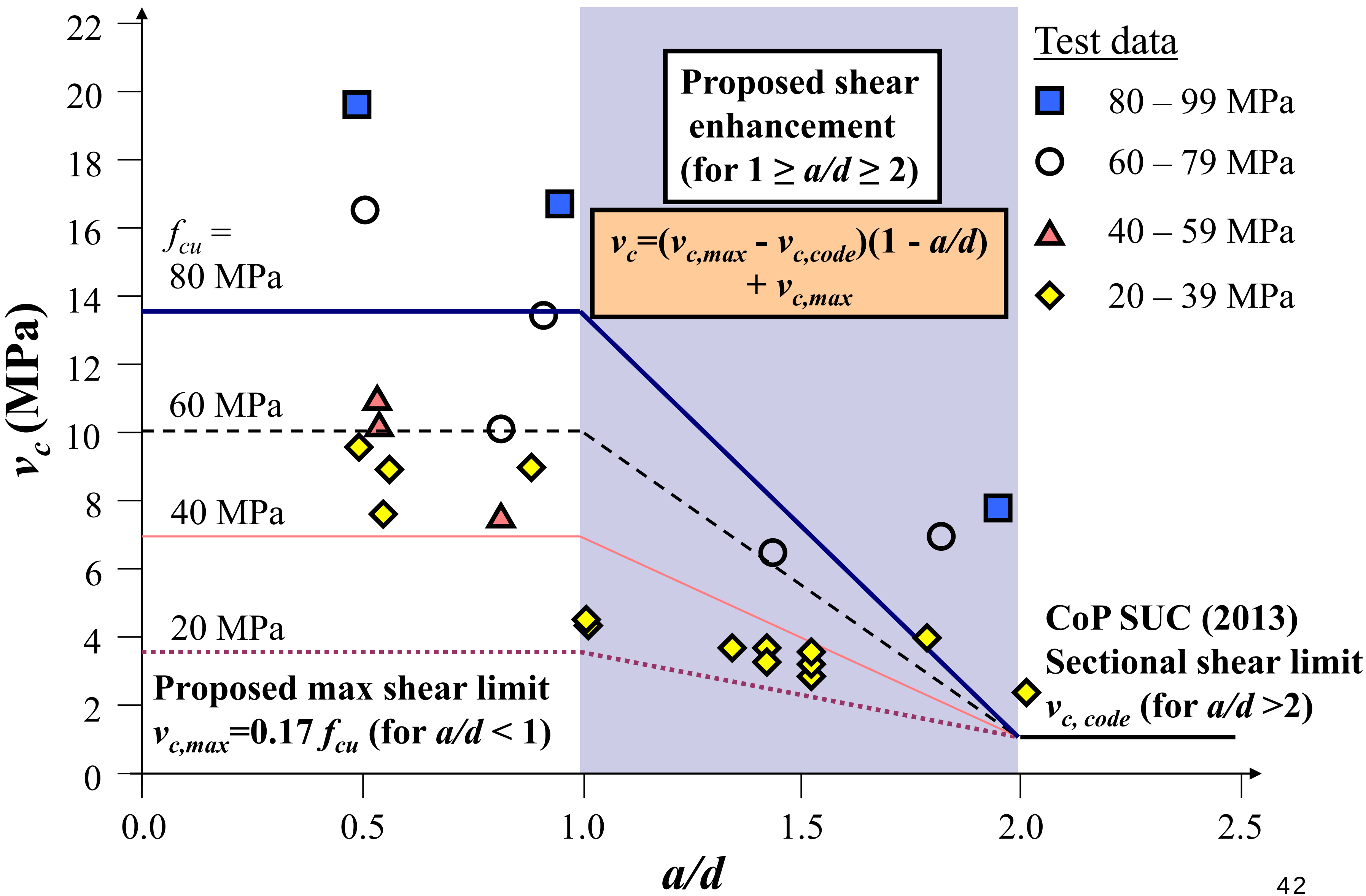
Group 2 Specimens	v_c/f_{cu}	$v_c/\sqrt{f_{cu}}$
a/z=1 or 45° (database)	0.19	0.97
	0.18	0.94
	0.21	1.34
	0.21	1.32
	0.21	1.61
(HKU)	0.26	1.54
	0.26	2.13
	0.23	2.31
a/z= 1/√3 or 60° (database)	0.23	1.40
	0.18	1.12
	0.20	1.38
	0.19	1.41
(HKU)	0.25	1.47
	0.29	2.31
	0.27	2.62
SD	0.0338	0.495
Mean	0.22	1.59
Coeff. of Variation	0.153	0.311
The lowest value	0.18	0.94

f_{cu} correlates better (almost double!)

Suggested lower bound shear limit
$v_c \leq 0.17\, f_{cu}/\gamma_m$

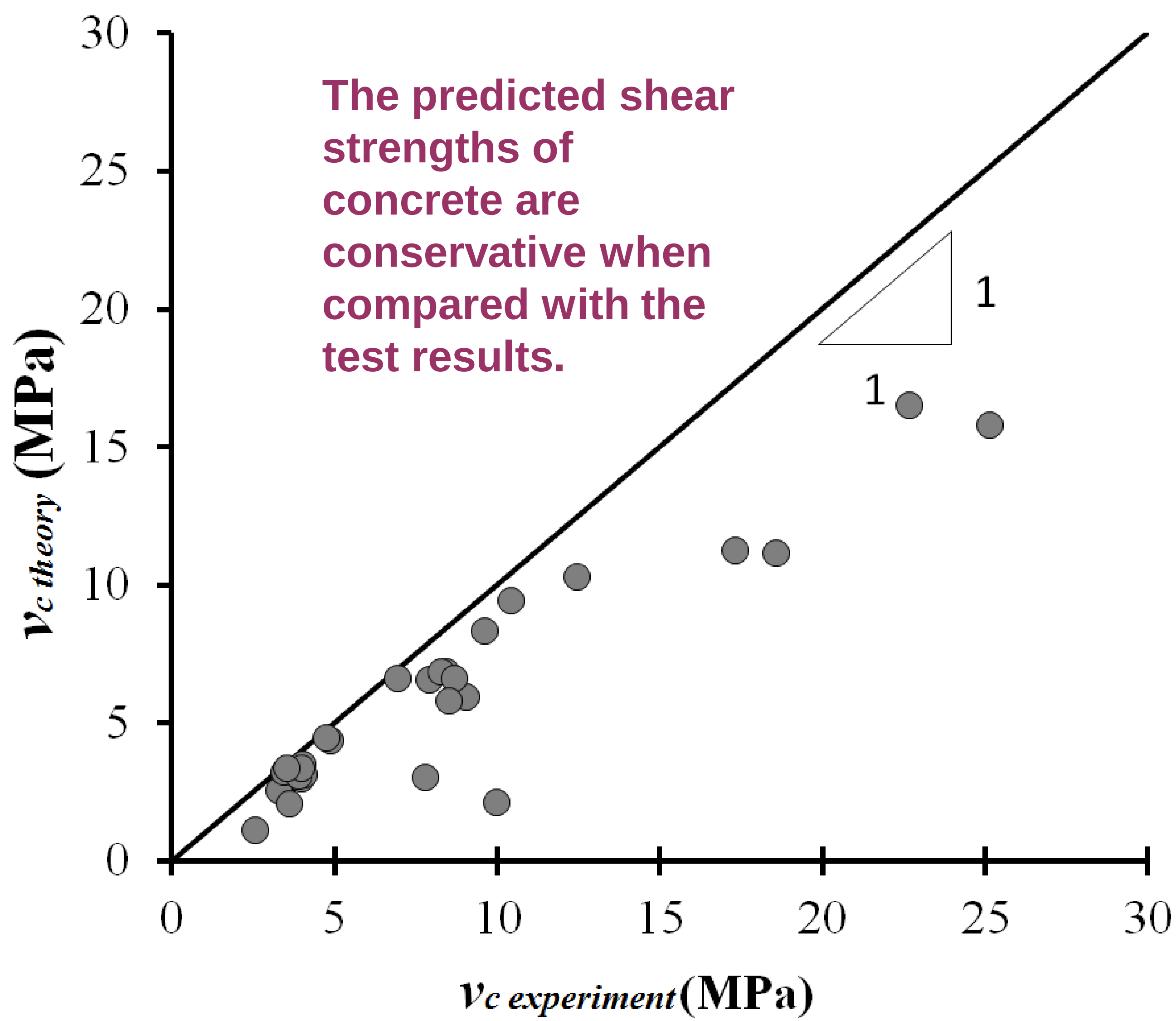
Enhancement of Shear Strength

Without material partial safety factor



Max Shear Strength Limit

Comparison between experimental and proposed theoretical results



Shear Stress Limit Model

Proposed design shear strength of concrete

$$v_c = v_{c\,code} / \gamma_{shear} \leq 0.17 f_{cu} / \gamma_{shear}$$

Shear enhancement when 1<*a/d* ≤ 2

$$v_c = \left[\left(0.17 f_{cu} - v_{c\,code} \right) \left(1 - a/d \right) + 0.17 f_{cu} \right] / \gamma_m$$

Concluding Remarks

- The design, characteristic and mean compressive strengths of concrete have been reviewed. A equation has been proposed to calculate the mean compressive strength of NSC.
- The full range stress-strain curve of unconfined concrete with the concrete strengths varying from 30 MPa to 90 MPa has been presented.
- A new shear strength model of concrete has been briefly discussed.
- More local concrete tests should be done to evaluate the basic mechanical properties of concrete, e.g. stress-strain curve of confined concrete, mean strength of HSC, etc. as well as the long term mechanical properties of concrete.

Thank you for Attention!

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